Warpage of Injection-Molded Thermoplastics Parts: Numerical Simulation and Experimental Validation

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A mathematical model and the integrated simulation program for predicting part warpage, which arises during injection molding of thermoplastic polymers, were developed. To build the model, the finite element method (FEM) was used and the theory of shells, represented as an assembly of flat elements formed by combining the constant strain triangular element and the discrete Kirchhoff triangular element, was applied. This shell theory is well suited for thin injection molded products of complex shape. Furthermore, in this work, experimental research on flat plates was performed to study the effect of the plastic material, the mold structure, and certain key processing parameters on warpage. The investigated processing parameters include injection pressure, packing pressure, injection time, packing time, cooling time, and melting temperature. Different examples were presented to examine the developed simulation software, and results predicted from the simulation program were verified in the above experiments and showed reasonable accuracy compared with experiment data.

Keywords experimental validation, injection molding, numerical simulation, warpage

1. Introduction

There are many kinds of part defects: shorts, flash, warpage, and weld lines during injection molding of thermoplastics.[1] Among these defects, warpage is the worst because it can cause a failure to a poor assembly or mating of joints and surface quality. Therefore, injection molders and customers now pay increasing attention to part warpage, which becomes a determining quality criterion for part acceptance. Nowadays, computer-aided engineering (CAE) in the injection molding process is used widely. Mold-filling simulation software[2] have now become useful tools of the mold designer and process engineer in addressing such issues as gate placement, runner sizing, and clamp-force requirements to avoid shorts, flash, and weld lines. Mold-cooling programs[3] can assist in designing the cooling system to reduce the whole cycle time to improve productivity and obtain uniform cooling for good part quality. Still, the prediction of warpage of the molded part is a challenging task for CAE applied to the injection-molding process. For this reason, many researchers have devoted themselves to study it by the aid of computer. A brief description of warpage in injection molding is given below.

St. Jacques[4] simulated thermal warpage in flat plates of an amorphous polymer due to unbalanced cooling. Its thermal warpage was predicted from calculated asymmetrical temperature profiles by using pure bending theory. Tamma and Railkar[5] proposed the transfinite element formulations for predicting the thermally induced warpage and stresses. Matsuoka and Takabatake[6] used a simple elastic model combined with the integral analysis of the whole-injection molding process to predict warpage of fiber-reinforced parts. Chiang, Himasekhar, Santhanem, and Wang[7] and Santhanam and Wang[8] used both an LRW (proposed by Lee, Rogers, and Woo[9])-based model and a perfect-fluid elastic model to predict shrinkage and warpage of injection molded parts. Akay and Ozden[10] considered unbalanced thermal residual stresses as the main cause of warpage and introduced empirical relationships to calculate the imbalance of residual stresses and the warpage of a flat plate. Kabanemi et al.[11] developed a three-dimensional viscoelastic model to predict the residual stresses and the shape of injection-molded products.

In this work, a numerical model was proposed based on the theory of shells as an assembly of flat elements and finite element method (FEM), and then the integrated simulation program was developed as a tool to predict part warpage for the purpose of providing proper dimensional tolerances, which is based on simulation outputs of filling, packing, cooling, orientation, and stress analysis. Moreover, a particular focus was made on investigating the effect of plastic material, mold structure, and key processing parameters of warpage. The integrated program was applied to high-density polyethylene (HDPE) square plate and polystyrene (PS) square plate. Warpage predicted from the simulation program was verified by the experimental data, which showed that theoretical prediction for the warpage correlated sufficiently with the experimental data qualitatively and quantitatively.

2. Theoretical Analysis

During the injection molding process, many kinds of stresses, including flow-induced residual stress, thermally induced residual stress, and residual stress caused by ejection force are generated. Consequently, the governing equation is represented as a function of local strain and all these residual stresses:

$$\{\sigma\} = [D]\,(\{\varepsilon\} + \{\varepsilon_0\}) + \{\sigma_0\} + \{\sigma_l\} + \{\sigma_t\} \qquad \text{(Eq 1)}$$

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where $\{\sigma\}$ is the stress, $\{\varepsilon\}$ is the strains, $\{\varepsilon_o\}$ represents thermal strain due to the subsequent cooling of parts after being ejected from the mold, $[D]$ is elasticity matrix, $\{\sigma_t\}$ denotes thermal residual stress, $\{\sigma_l\}$ is flow residual stress, and $\{\sigma_o\}$ is the residual stress caused by other reasons such as ejection force.

By selecting an admissible displacement field, displacements within any element can be interpolated from the nodal degree of freedom of that element. Nodal displacements and curvatures for shell elements are expressed as follows:

$$\{r_e\} = [k]\{d\} = \int [B_e]^T [D][B_e] dv_e \{d\} \tag{Eq 2}$$

where $[k]$ is the element stiffness matrix, and the element load vector can be evaluated by:

$$\{r_e\} = \int [B_e]^T [D]\{\varepsilon_0\} dv_e - \int [B_e]^T \{\{\sigma_0\} + \{\sigma_l\} + \{\sigma_t\}\} dv_e \tag{Eq 3}$$

where $[B_e]$ is the derivative operator matrix.

It is true that using a full three-dimensional FEM analysis can achieve very accurate warpage results; however, it is cumbersome when the part to be analyzed is in complicated shape. Therefore, a two-dimenstional FEM model, based on shell theory, is used because the thickness of an injection-molded part is much smaller than the dimensions along the part's surface and the part can be regarded as an assembly of flat elements to predict part warpage.[12] The three node shell elements consist of the constant strain triangular element (CST) and the discrete Kirchhoff triangular element (DKT) as shown in Fig. 1. Thus, part warpage can be separated into plane-stretching deformation and plate-bending deformation, and the warpage element stiffness matrix can also be divided into the stretching-stiffness matrix and bending-stiffness matrix. More important, this kind of element permits use of simulation results for filling, packing, cooling, fiber orientation, and stress analysis based on the same finite element meshes and thus eliminates considerable computation cost.

By definition, strain is obtained from displacement by differentiation. Thus the in-plane strain-displacement relation and the plane element stiffness matrix $[k_p]$ can be evaluated by the following:

$$\{\varepsilon_p\} = [B_p]\{d_p\} \tag{Eq 4}$$

$$[k_p] = \int [B_p]^T [D_p][B_p] dA \tag{Eq 5}$$

where $[B_p]$ is a derivative operator matrix and $\{d_p\}$ is the in-plane displacement vector including two translational degrees and one rotational degree of freedom on each node as shown in Fig. 1(a).

Similarly, the bending element stiffness matrix can be formed using the same procedure. The rotational curvature, $\{\varepsilon_b\}$, can be represented in terms of bending displacement transformation matrix $[B_b]$ and displacement matrix $\{d_b\}$ as shown in Fig. 1(b), and bending element stiffness matrix $[k_b]$ can also be evaluated as:

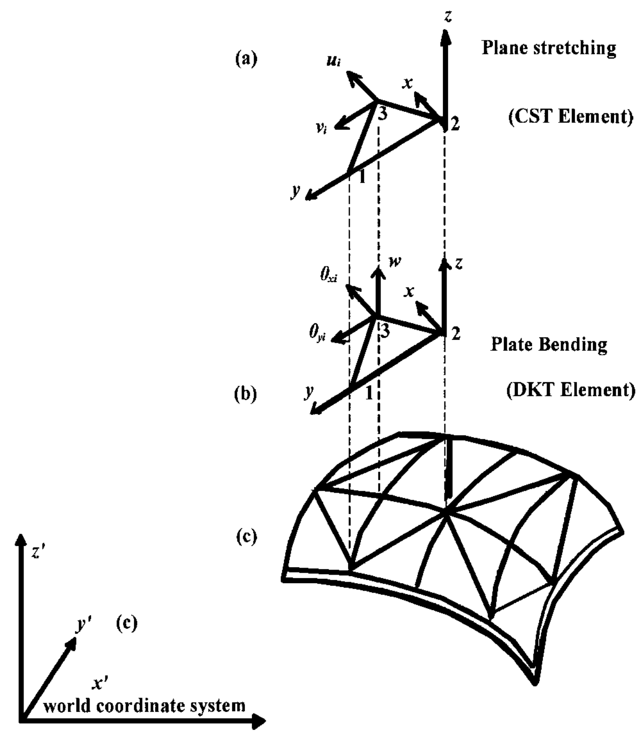


Fig. 1 Deformation decomposition of shell element in the local coordinate system. **(a)** in-plane stretching element; **(b)** plate-bending element; **(c)** shell element

$$\{\varepsilon_b\} = [B_p]\{d_b\} \tag{Eq 6}$$

$$[k_b] = \int [B_b]^T [D_b][B_b] dA \tag{Eq 7}$$

Thus, an in-plane stiffness sub-matrix $[k_p']$ and a bending stiffness sub-matrix $[k_b']$ can be combined into shell stiffness sub-matrix $[k']$ without mutual interaction.

$$[k'] = \begin{bmatrix} k_p' & 0 & 0 \\ 0 & k_b' & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{Eq 8}$$

Illustrated as the above, because the shells consist of an assembly of flat elements, a difficulty arises if all the elements meeting a mode are coplanar. In that case, Eq 8 indicates that the structure stiffness matrix will be singular. An easy solution is to change the diagonal coefficient from zero to a nonzero number because θ_z is unloaded and not connected to any other degree of freedom.

As to the boundary condition of the FEM model, it is different when the part is molded in the injection mold and ejected from the mold. In the mold, the product cannot shrink or warp freely because of the constraint of the mold core. This interaction between the mold and the polymer is taken into consideration by preventing the out-of-plane displacement. Therefore, only in-plane displacements are possible. After the part is ejected from the mold, free shrinkage and warpage are allowed.

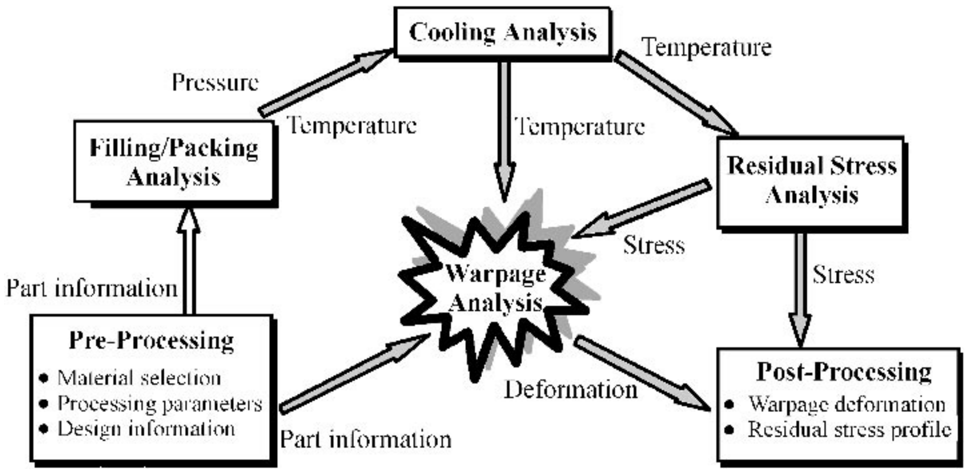


Fig. 2 Integrated simulation program for the prediction of warpage

Thus, both the in-plane and out-of-plane displacements are computed.

3. Integrated Simulation

Many mechanisms cause part warpage: uneven filling, uneven packing, uneven cooling and, thus, uneven residual stresses.[13] Consequently, it is necessary to predict warpage based on simulation outputs at the end of the molding cycle, including filling, packing, cooling, orientation, and stress. The integrated computational flow chart of warpage is illustrated in Fig. 2. After the user selects both the polymer and mold material, and finishes the mold design (including runner and gate design, cooling system design), filling/packing analysis is applied toward solving the temperature and pressure profiles. Those temperatures and pressures are used as the boundary conditions, and thus a cooling program can be applied to calculate the temperature of both the mold and the part. Based on the temperature and pressure profiles, residual stress analysis software can output the calculated thermal-induced residual stresses. Because those residual stresses are calculated as input data, warpage analysis can be applied, and displacement of each node in the part can be shown in several ways. Consequently, this integrated simulation program can provide a tool to predict the warpage of the molded part during the design stage without actual tooling and molding operation.

4. Experiments

In the experiment, investigation has been implemented from the following three aspects[14] that influence part warpage most: material (PS and HDPE); mold gate (wide fan gate and narrow rectangular gate); and key processing parameters (injection pressure, packing pressure, cooling time, injection time, packing time, and melting temperature). The materials used for the experiment are amorphous PS and crystalline HDPE. Their main mechanical specifications are summarized in Table 1.

The experimental plate is a rectangular slab with the dimensions of 200 × 40 × 4 mm. The mold cavity is designed to be an exchangeable gate structure. One is a wide fan gate of 36 × 1 mm; the other is a narrow rectangular gate of 8 × 1 mm, as shown in Fig. 3 and 4.

To investigate the relationship between part warpage and each key processing parameter, the concept for deliberately varying one variable and maintaining all the other variables constant was applied in this experiment. For example, in one set of experiments as shown in Table 2, all other parameters except cooling time were maintained constant, and cooling time varied from 10 to 50 s according to Table 3 to study its effect on warpage.

The injection-molding experiments were performed on a servo-controlled injection-molding machine (CJ800-400NC Zhengde) with a clamping force of 350 KN. After injection molding, the flat plates were stored for 48 h at 26 °C. The warpage of plate was measured by micrometer with the accuracy of 0.01 mm, and the measured maximum warpage is shown in Fig. 5. To improve the accuracy of the experiment, five samples were kept and measured, and the averaged maximum warpage was recorded for each molding condition.

5. Validation and Discussion

The first example was to verify the predicted warpage of HDPE plate with a wide fan gate under high packing pressure. As shown in Table 4, the packing pressure varied and all other process parameters maintained constant. The normal packing pressure used in the experiment was 72 Mpa, and in this case, the HDPE plate maintained flat and did not warp. To investigate the effect of packing pressure on warpage, packing pressure was deliberately varied from 72 Mpa to 103 Mpa. When the packing pressure was set at 101 Mpa, the measured maximum warpage was 3.27 ± 0.01 mm, and the predicted warpage was calculated as 2.67 mm. The original mesh of HDPE plate with a wide fan gate was shown as Fig. 6(a) and showed no warpage, and its deformed mesh amplified by a factor of 2 was

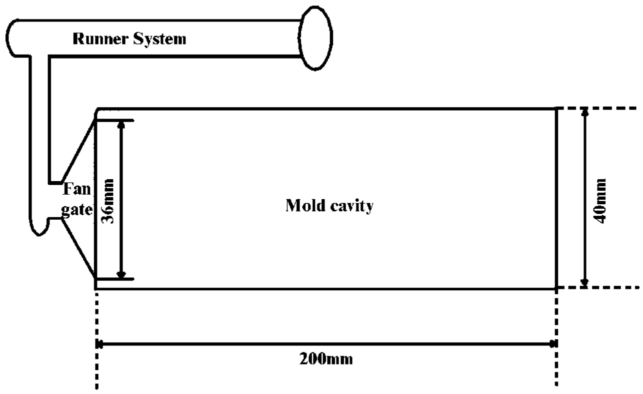


Fig. 3 Cavity and runner system with fan gate

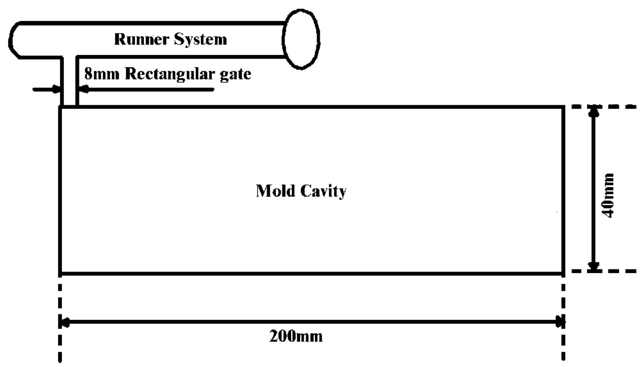


Fig. 4 Cavity and runner system with square gate

Table 1 Material Properties of PS and HDPE

Material	Supplier	Elastic Modulus (MPa)	Poisson's Ratio
PS	Yanshan Petrochemical Co.	950	0.38
HDPE	Hangzhou Petrochemical Co.	3500	0.32

Table 2 Constant Process Parameters of One Set of Experiments

Gate: Wide fan gate			Material: Amorphous PS		
Constant Parameters	**Injection Pressure**	**Packing Pressure**	**Injection Time**	**Packing Time**	**Melting Temperature**
Values	57.6 Mpa	72 Mpa	6 s	3 s	165

shown in Fig. 6(b). It was observed that the trends of both the experimental and the predicted warpage were very close and showed a cup-like shape, although there was a 0.6 mm difference between them.

The second experiment was to verify the predicted warpage of HDPE plate with a narrow gate under a high melting temperature. According to the process condition shown in Table 5, when the variable melt temperature was set to be 205 °C, the measured maximum warpage of HDPE plate was 2.28 ± 0.01 mm. In this circumstance, the calculated warpage of the HDPE plate by using the simulation program was 1.35 mm. It was obvious that the predicted warpage was much less than the measured warpage, yet they both showed the same cup-like deformed shape.

To investigate the effect of packing time on part warpage, a PS plate molded with a wide fan gate was injected under a long packing time, which was 2 s in normal case and would not cause the plate warp. As shown in Table 6, the packing time was deliberately added up to 20 s, and the measured maximum warpage of the PS plate was 0.22 ± 0.01 mm. As to the predicted warpage, the calculated value was 0.18 mm and was almost the same with the measured warpage (Table 6). It can be seen that the trends of both the experimental and the predicted warpage were very close and both showed a cup-like shape.

The fourth example examined the effect of long cooling time on the warpage of a PS plate with a narrow gate. All the constant parameters were shown in Table 7, and the variable parameter (cooling time) was deliberately set to be 40 s. The measured maximum of PS plate was 0.82 ± 0.01mm, yet the deformation calculated by the integrated program was 0.44 mm and less than the measured warpage.

From the above four comparison examples, the conclusion can be drawn that the predicted warpage agrees well with the experimental warpage both quantitatively and qualitatively. In detail, the predicted displacements were less than experimental

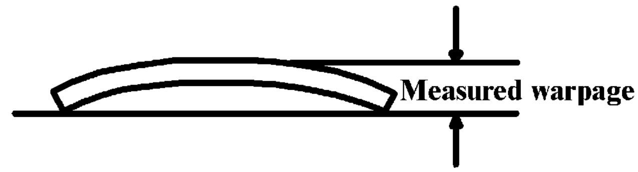


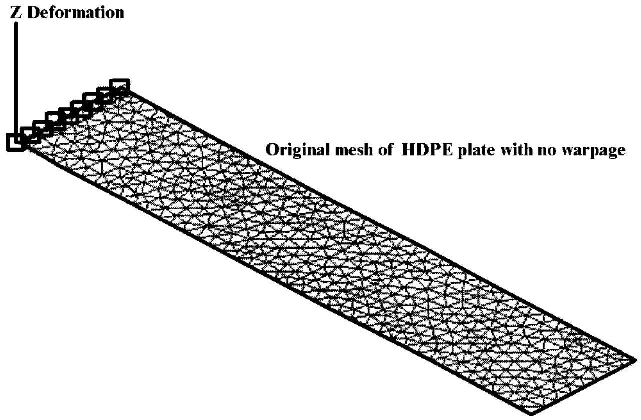
Fig. 5 Measured maximum warpage for one part

Table 3 Varying Parameters–Cooling Time of One Set of Experiments

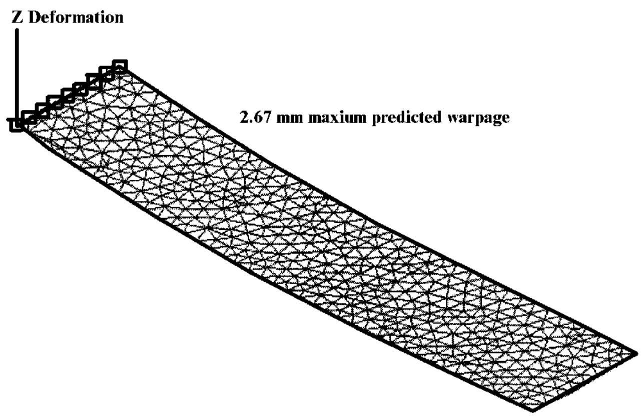
Gate: Wide Fan Gate				Material-Amorphous PS					
Cooling Time (s)	10	15	20	25	30	35	40	45	50

Table 4 Measured Warpage and Predicted Warpage Under High Packing Pressure

HDPE Plate with Wide Fan Gate	
Constant parameters	
Injection pressure	51.8
Mpa Injection time	3 s
Packing time	3 s
Cooling time	50 s
Melt temperature	165 °C
Air temperature	26.3 °C
Average mold temperature	48.4 °C
Variable parameters	
Packing pressure	
Measured warpage:	3.27 mm
Simulated warpage:	2.67 mm



(a)



(b)

Fig. 6 Predicted warpage of HDPE plate with a wide fan gate under 101Mpa packing pressure. **(a)** Original mesh of HDPE plate with no warpage; **(b)** warpage amplified by a factor of 2

Table 5 Measured Warpage and Predicted Warpage Under High Melting Temperatures

HDPE Plate with Narrow Gate	
Constant parameters	
Injection pressure	54.7 Mpa
Packing pressure	72 Mpa
Injection time	6 s
Packing time	3 s
Cooling time	50 s
Air temperature	28 °C
Average mold temperature	54.8 °C
Variable parameters	
Melt temperature	
Measured warpage:	2.28 mm
Simulated warpage:	1.35 mm

warpages, and the simulated warpage for PS plate was closer to the experiment data than that of HDPE plate. There are two possible reasons for this discrepancy. Residual stress analysis neglects nonlinear visco-elasticity and flow induced residual stress, so the predicted residual stress and warpage are inevitably smaller than the actual stress and deformation. Moreover, cooling analysis software assumes the material data independent of temperature and does not consider the crystallization behavior of the polymer[1,13]; therefore, the discrepancy is larger for a polymer that experiences crystallization such as HDPE.

Table 6 Measured Warpage and Predicted Warpage Under Long Packing Times

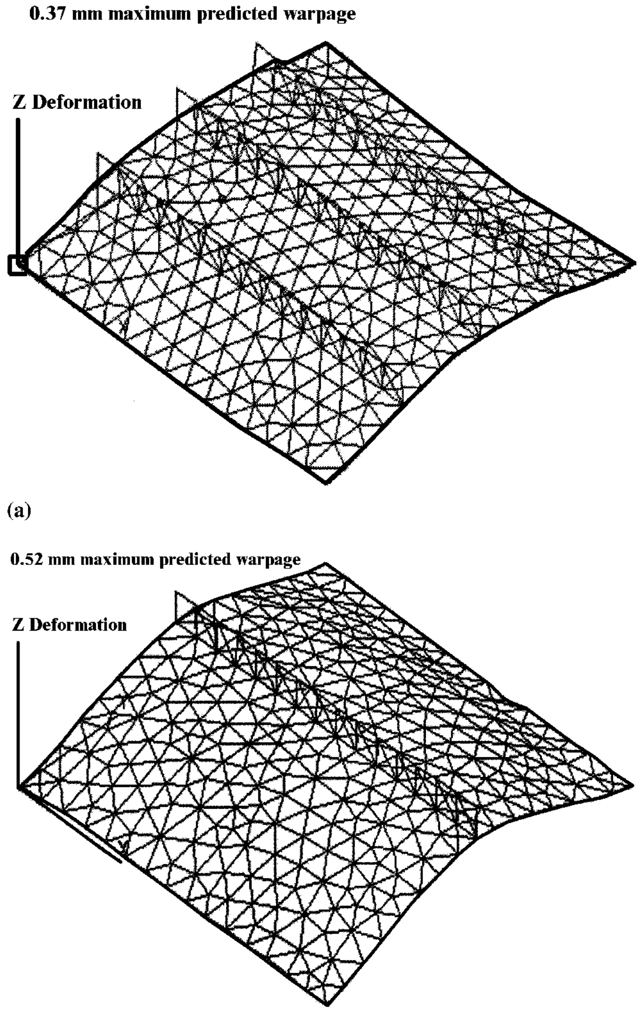
PS Plate with Wide Fan Gate	
Constant parameters	
Injection pressure	57.6 Mpa
Packing pressure	72 Mpa
Injection time	6 s
Cooling time	30 s
Melt temperature	170 °C
Air temperature	27.4 °C
Average mold temperature	34.2 °C
Variable parameters	
Packing time	
Measured warpage:	0.22 mm
Simulated warpage:	0.18 mm

Table 7 Measured Warpage and Predicted Warpage Under Long Cooling Times

PS Plate with Wide Fan Gate	
Constant parameters	
Injection pressure	50.4 Mpa
Packing pressure	72 Mpa
Injection time	6 s
Packing time	2 s
Melt temperature	165 °C
Air temperature	29.1 °C
Average mold temperature	47.9 °C
Variable parameters	
Cooling time	
Measured warpage:	0.82 mm
Simulated warpage:	0.44 mm

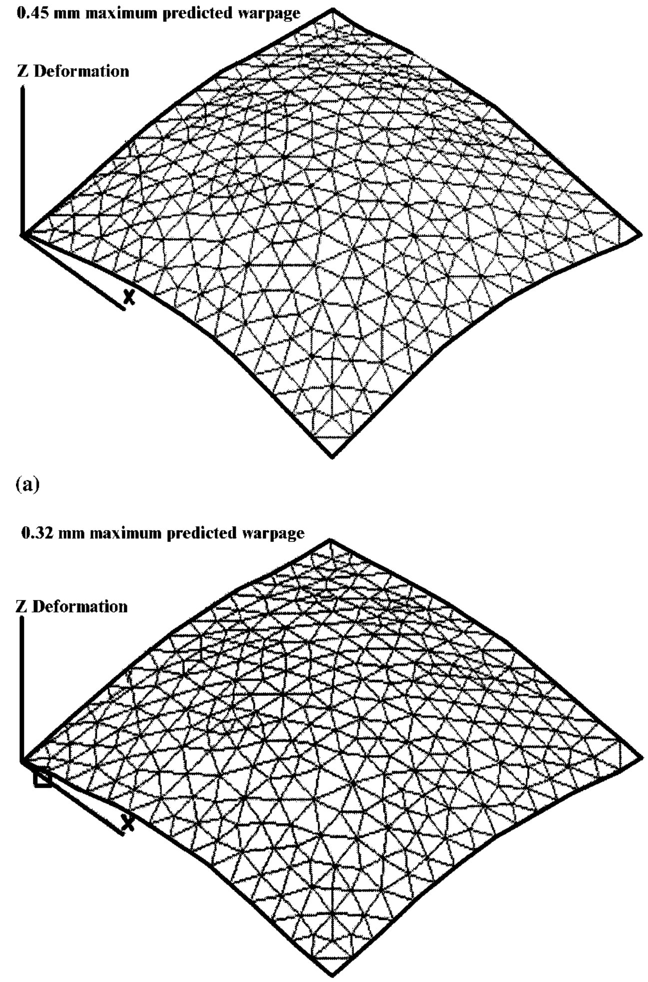
The simulation program was further verified by the following examples. Figure 7 showed warpage of two kinds of 2 mm thick plate with various ribs. One PS plate had three ribs, and another PS plate had only one rib. The processing parameters of those two plates were the same. The injection pressure was 40 Mpa, the packing pressure was 52 Mpa, the injection temperature of melt polymer was 180 °C, and both of them were cooled under the same cooling process condition. However, their predicted warpage differed because these two parts had different geometrical structures. As shown in Fig. 7, predicted warpage of the plate with three ribs was 0.37 mm, smaller than that of the plate with one rib (0.52 mm). The main possible reason is that ribs improve the stiffness of plate and the ability to resist deformation.

Another example examined was the effect of part thickness on warpage. Figure 8 illustrates deformation of 2 mm thick PS plate and 3 mm thick PS plate (amplified by a factor of 15) separately. The processing parameters of these two plates were the same as the above. The predicted warpage of the thinner plate was 0.45 mm, and the calculated deformation of the thicker one was only 0.32 mm; the thinner plate warped more than thicker plate after molding. This is because



(b)

Fig. 7 Effect of structure on warpage of parts (amplified by a factor of 30). **(a)** Warpage of three-ribbed square plate; **(b)** warpage of square plate with one rib



(b)

Fig. 8 Effect of thickness on warpage of plate (amplified by a factor of 15). **(a)** Warpage of 2 mm thin plate; **(b)** warpage of 3 mm thick plate

that stiffness of a thick plate is larger than that of a thin plate, and thus it is more difficult to warp than a thin plate. This predicted phenomenon of the simulation software well agrees with the shell theory.

6. Conclusion

A mathematical model to describe warpage of injection-molded parts was presented on the basis of shell theory, and it was numerically solved by use of FEM. To verify the predicted warpage, experiments were performed to study the effects of plastic material, mold geometry, and several kinds of processing parameters on warpage. Results predicted from the simulation program were verified with experimental data. Both the qualitative and quantitative results for the theoretical prediction correlated sufficiently with the experimental values. Of course, there are certain differences between the predicted warpage and the measured warpage. Consequently, many improvements are necessary. In the theoretical model, the molecular orientation, accurate material data, and crystallization behavior should be added to the integrated simulation of flowing, packing, cooling, and stress analysis in future studies.

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